Geometry Notes

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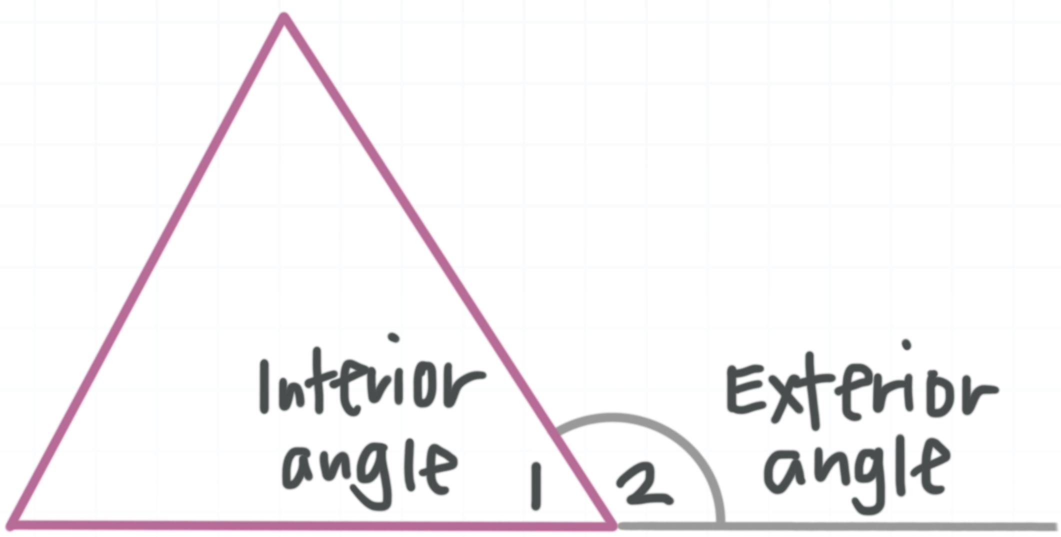
MATH

Exterior angles of polygons

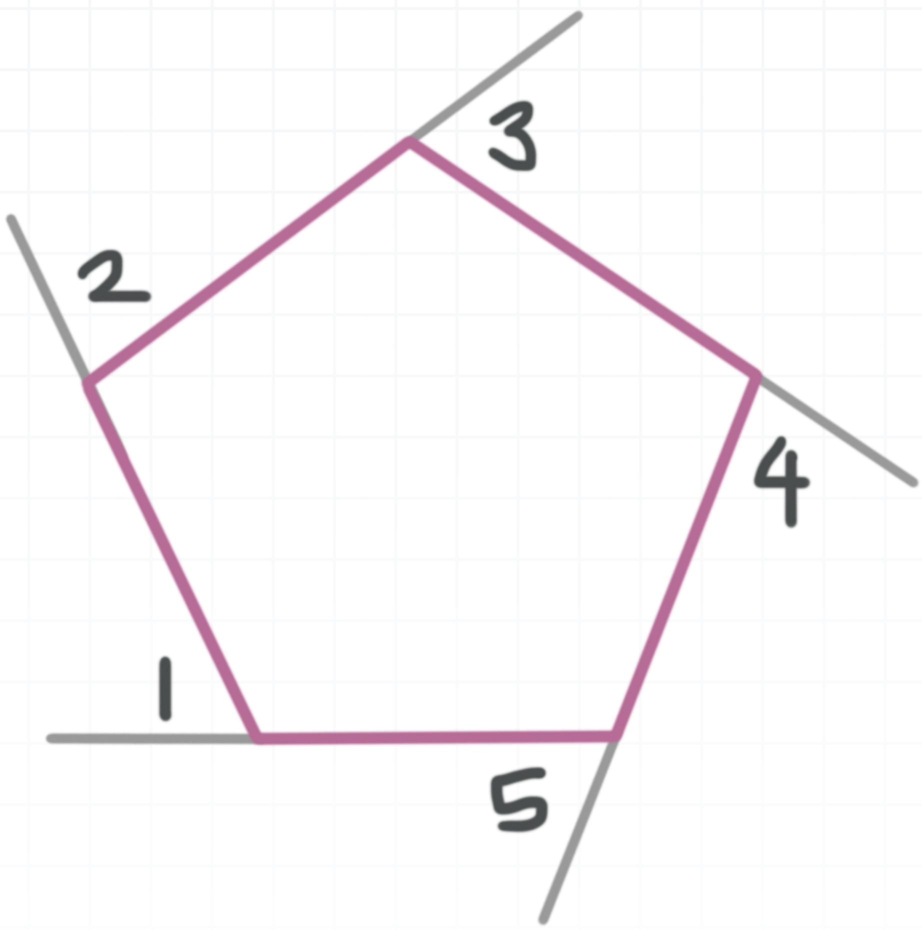
In this lesson we'll look at exterior angles of polygons and the relationship between those and their corresponding interior angles.

The exterior angle is the angle formed on the outside of a shape from a line extended from the next side.



The interior angle and the exterior angle are supplementary. This means $m\angle 1 + m\angle 2 = 180^\circ$.

The sum of all of the exterior angles in any polygon is always 360°. Here is an example of the exterior angles of a pentagon adding to 360°.

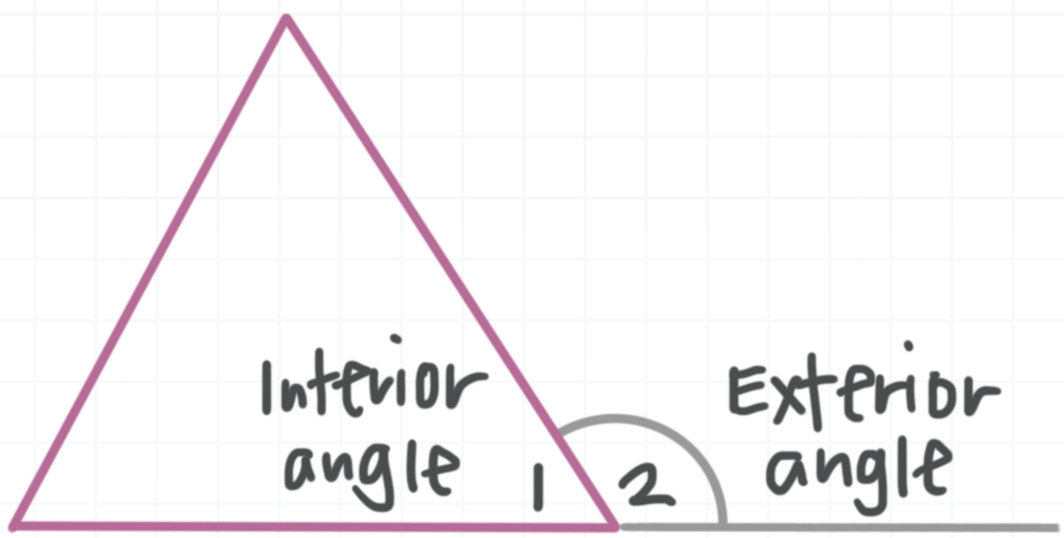


$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$$

Remember also that the interior angles of any polygon always sum to $(n-2)180^\circ$.

Example

Given that the triangle in the diagram is equilateral, what is the measure of angle 2?



The interior angles in a triangle add to 180°. An equilateral triangle is also equiangular. That means each angle measures $180^\circ \div 3 = 60^\circ$, so $m\angle 1 = 60^\circ$. $\angle 1$ and $\angle 2$ are supplementary, which means

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$60^\circ + m\angle 2 = 180^\circ$$

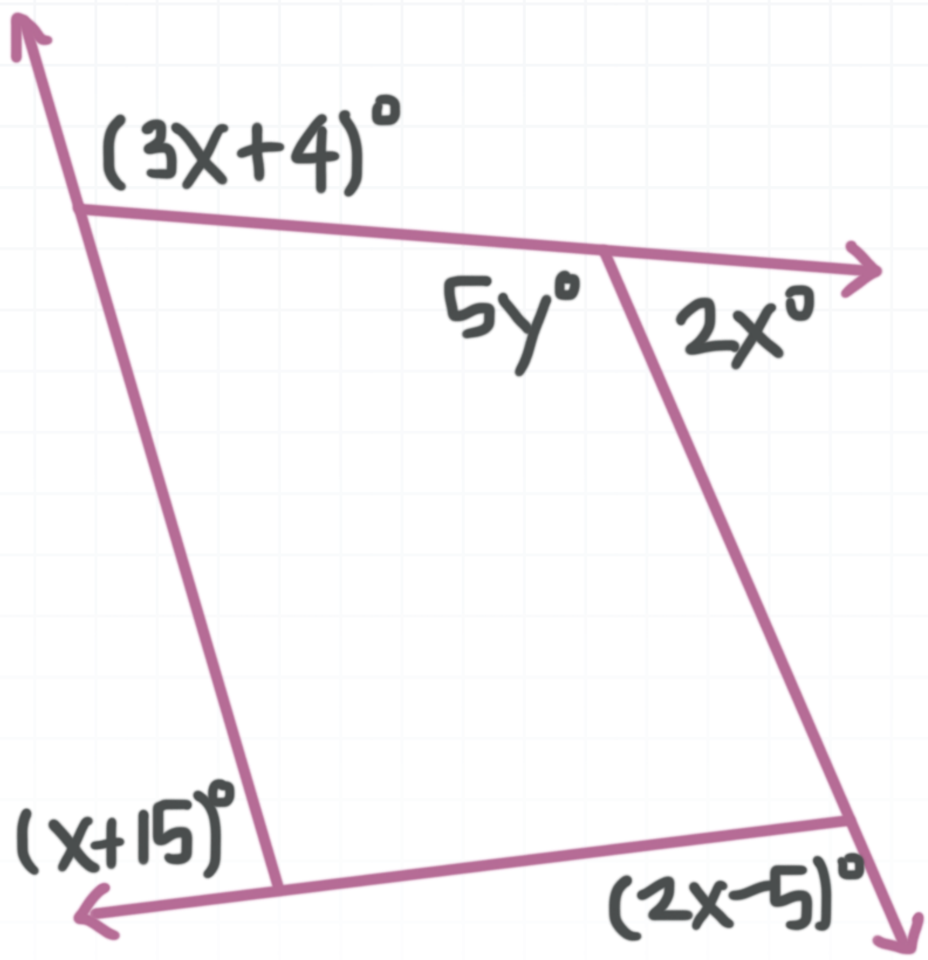
$$m\angle 2 = 120^\circ$$

We could also have solved this problem by using the fact that all of the exterior angles sum to 360°. A triangle has three interior angles, so it also has three exterior angles. Since all of the interior angles are congruent, all of the exterior angles will also be congruent. This means $m\angle 2 = 360^\circ \div 3 = 120^\circ$.

Let's look at a few more example problems.

Example

Find the value of y.



The sum of the exterior angles must be 360°. Therefore, we can say

$$3x^\circ + 4^\circ + 2x^\circ + 2x^\circ - 5^\circ + x^\circ + 15^\circ = 360^\circ$$

$$3x^\circ + 2x^\circ + 2x^\circ + x^\circ + 4^\circ - 5^\circ + 15^\circ = 360^\circ$$

$$8x^\circ + 14^\circ = 360^\circ$$

$$8x^\circ = 346^\circ$$

$$x^\circ = 43.5^\circ$$

$5y^\circ$ and $2x^\circ$ make a straight line so they are supplementary. Substitute 43.5° for x and solve for y.

$$5y^\circ + 2x^\circ = 180^\circ$$

$$5y^\circ + 2(43.25^\circ) = 180^\circ$$

$$5y^\circ + 86.5^\circ = 180^\circ$$

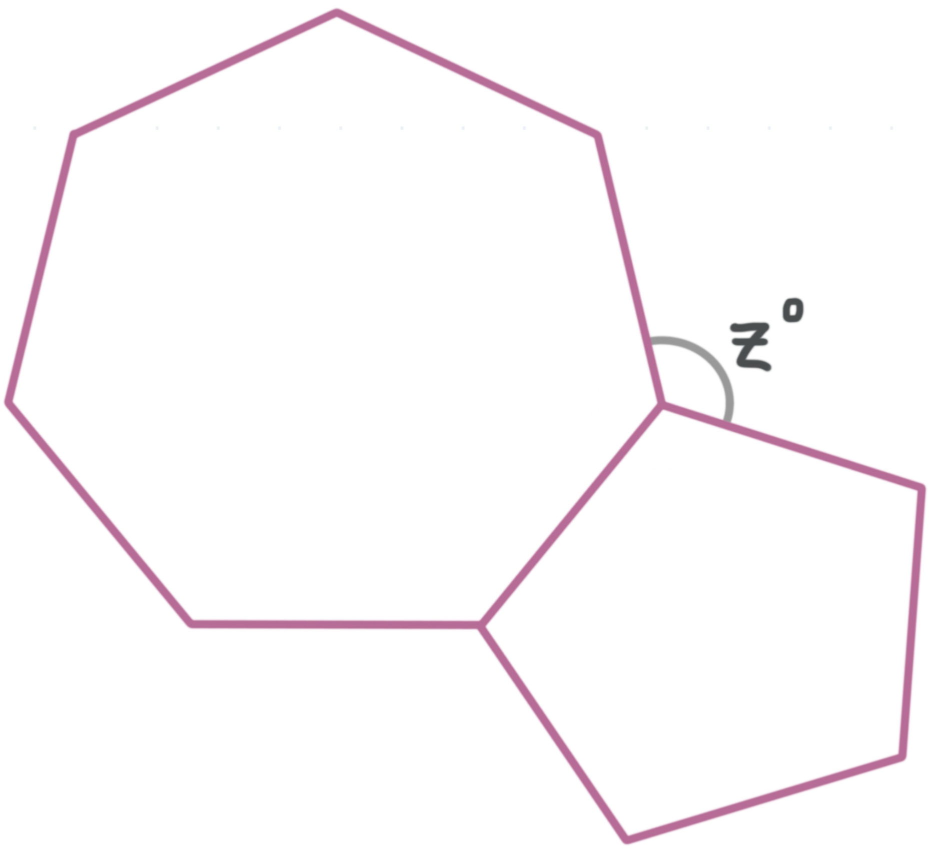
$$5y^\circ = 93.5^\circ$$

$$y^\circ = 18.7^\circ$$

Let's look at one more example problem.

Example

The figure shows a regular heptagon and a regular pentagon. Find the value of z to the nearest hundredth.

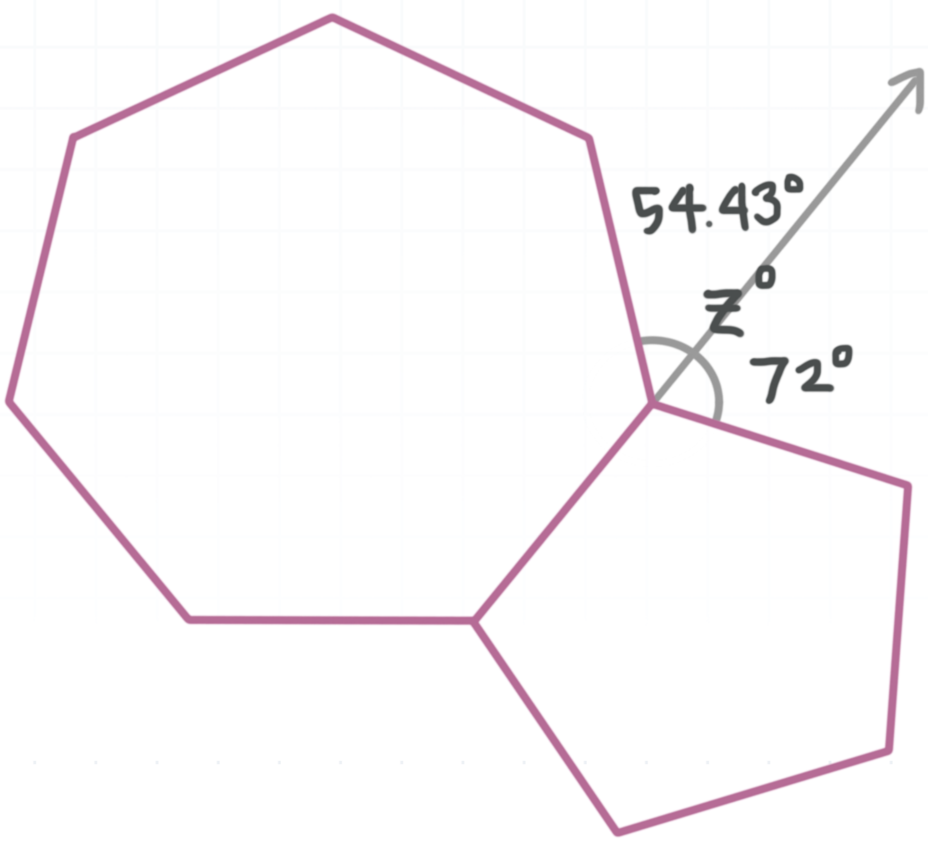


An exterior angle of a regular heptagon has a measure of

$$360^\circ \div 7 \approx 51.43^\circ$$

An exterior angle of a regular pentagon has a measure of

$$360^\circ \div 5 = 72^\circ$$



Therefore we can say that z is given by

$$z^\circ \approx 51.43^\circ + 72^\circ \approx 123.43^\circ$$

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